LINEAR POTENTIALS IN THE CORES OF CLUSTERS OF GALAXIES

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Abstract: We make a first application of the linear gravitational potentials of the conformal gravity theory to the distance scale associated with clusters of galaxies, with the theory being found to give a reasonable value for the mean velocity of the virialized core of the typical Coma cluster, with no need to invoke the existence of dark matter.

In a recent series of papers (Mannheim 1994 gives a full bibliography) Mannheim and Kazanas have explored conformal invariant fourth order gravity as a candidate alternative to the standard second order Newton-Einstein theory. In their initial paper they showed that in this theory the potential of a gravitational source takes the form $V(r) = -\beta c^2/r + \gamma c^2 r/2$, to thus both nicely recover the Newtonian potential and at the same time introduce a new source-dependent length scale γ which would then parameterize any possible departures from the standard phenomenology. Moreover, Mannheim (1993) then showed that with the use of this potential it is possible to fit the rotation curves of some typical galaxies without needing to assume the existence of any dark matter, with the contribution of the linear potential when integrated over the observed surface brightness data then completely replacing the dark matter contribution in the same galaxies. Intriguingly it was found from the fitting that the strength of the linear potential of an entire galaxy was typically of the order of the inverse Hubble length to suggest a possible cosmological origin for γ. Armed with an actual magnitude for the galactic γ we can now apply the conformal theory to the first available distance scale beyond galaxies, namely that of clusters of galaxies. Interestingly, we find that the conformal theory deviates from Newton there by just the amount needed to nicely accommodate a virialized cluster core without invoking dark matter, though the theory may turn out to have some difficulties should entire clusters prove to be virialized.

In applying the standard cluster virial analysis to theories with rising rather than falling potentials two problems arise which are not considered in the standard theory. First since the standard discussion generally considers the two-body collision dependent term in the Boltzmann equation to be a local perturbation on the global mean field set up by the gravitational field of the rest of the galaxies in the cluster we have to reexamine the entire formalism in light of the conformal gravity linear potentials which grow with distance and which can thus not be thought of as producing localizable collisions at all. Moreover, in a theory with rising potentials, we are not free to ignore the effects due to all of the rest of the galaxies in the Universe, so that even if a system such as a cluster of galaxies is geometrically isolated, that does not immediately mean that it is gravitationally isolated in our theory or that it is bound purely under its own self forces. As regards the inapplicability of the Boltzmann analysis within clusters in the linear potential case, we note that fortunately there is an altogether far more general statistical analysis beyond Boltzmann, namely that based on the Liouville equation as conveniently treated via the very general Bogoliubov, Born, Green, Kirkwood, and Yvon (BBGKY) heirarchy (a heirarchy which reduces to the Boltzmann equation only under very special circumstances and which remains valid even when no such reduction is possible). Since the BBGKY hierarchy is valid for any central potential it is thus immediately valid in the linear potential case too, with our ability to apply the standard Jeans and Vlasov virial equations then reducing to the degree to which we can ignore two-particle BBGKY correlations (see Mannheim 1995 for details). Since the BBGKY relaxation time is currently unknown it is not immediately clear how much of a typical cluster has so far virialized (we argue below that possibly only the core is virialized), and so we develop below a formalism which can deal with both fully and partially virialized clusters. As regards the effect of the global coupling of the cluster to the rest of the Universe, this would require the developing of a theory for the growth of inhomogeneities and galaxy formation, with the relevant issue for motions within clusters then being not so much the coupling of each cluster to the general Hubble flow produced by a homogeneous background distribution of linear potential sources, but rather its coupling to the deviations from that flow caused by the presence of inhomogeneities. Since a theory for the growth of inhomogeneities in conformal gravity has yet to be developed, we are currently unable to address this issue explicitly, though as a first step we will determine approximately where it is that the cluster begins to merge with the general cosmological background, and should thus immediately anticipate that only those regions of the cluster which lie well within this merging radius are well enough isolated to be able (if then given enough time) to decouple and subsequently virialize.

To treat clusters in general let us consider a spherically symmetric cluster with matter volume density $\sigma(r)$ and matter surface density $I(R)$ contained within some volume of radius R_M, and let us suppose that only some inner kinematic region of the cluster within some virialization radius r_m has so far had time to virialize. Then within the $r < r_m$ region we may ignore all two-body correlations with the Jeans equation

$$\frac{d}{dr}(\sigma(r) < v_r^2 >) + \frac{2\sigma(r)}{r}(< v_r^2 > - < v_\theta^2 >) = -\sigma(r)V'(r) \tag{1}$$

then holding in this region. Despite the fact that Eq. (1) only involves points with $r < r_m$, we note that the potential $V(r)$ needed for it is obtained by integrating the Newtonian and linear potentials over the entire cluster and not just the virialized region, with the linear piece (unlike the Newton piece) actually receiving contributions from points with $r > r_m$. Now observationally we measure the two-dimensional projected line of sight velocity distribution average $< \sigma_p^2(R) >$ which is related to three-dimensional velocity averages (the ones which appear in the Jeans equation) according to

$$I(R) < \sigma_p^2(R) >= 2\int_R^{R_M} \frac{dr\sigma(r)}{r(r^2 - R^2)^{1/2}} \left(< v_r^2 > (r^2 - R^2) + < v_\theta^2 > R^2\right) \tag{2}$$

with the integration range all the way to the cluster radius R_M in Eq. (2) thus involving not only the desired virialized $r < r_m$ region but also the non-virialized $r > r_m$ region where Eq. (1) does not hold. However, integrating Eq. (2) itself over a sphere of radius r_m then yields (Mannheim 1995)

$$2\pi \int_0^{r_m} dRRI(R) < \sigma_p^2(R) >= \frac{4\pi}{3}\int_0^{r_m} drr^2\sigma(r)\left(< v_r^2 > +2 < v_\theta^2 >\right) \tag{3}$$

an exact and completely general relation in which the undesired unvirialized $r > r_m$ region has crucially been projected out. Since Eq. (3) only involves the region $r < r_m$, it thus only involves the region where we have presupposed the Jeans equation to be valid, with its use then yielding for the spatially averaged line of sight velocity

$$2\pi(\sigma_p^2(r_m))_{av}\int_0^{r_m} dRRI(R) = \frac{4\pi}{3}\int_0^{r_m} drr^2\sigma(r)rV'(r) \tag{4}$$

to give finally the form of the virial theorem for partially virialized systems.

For the most studied cluster, the Coma cluster, the relevant data may be found in White et al (1993) and references therein. At the distance of Coma an arc minute is $20/h$ kpc (for a Hubble parameter $H_0 = 100h$ km/sec/Mpc), so that the standard Abell radius is $75'$ for Coma. The surface brightness may be approximately fitted by a modified Hubble profile ($\sim 1/(R^2+R_0^2)$) with a core radius $R_0 = 9.23'$ and number density normalization $\sigma_0/R_0^3 = 0.016$ galaxies per cubic arc min., with the observed cluster data going out about to $3^\circ \simeq 20R_0 = R_M$ or so where we shall thus cut off the distribution. (A modified Hubble has to be cut off somewhere since it would otherwise yield an infinite total mass). White et al (1993) quote a total blue surface luminosity within the Abell radius of $L_B = 1.95 \times 10^{12}/h^2\ L_{B\odot}$, and a mean projected line of sight velocity of 970 km/sec for a convenient magnitude limited cut on the data which restricts to $R \leq 120'$. For such a mean velocity, the time required to cross the associated $240'$ diameter is $1.5 \times 10^{17}/h$ sec, which is of order $1/2H_0$, i.e. of order half a Hubble time, and thus we should not expect the entire cluster to have yet had time to virialize. Hence, for the purposes of this study, we shall simply assume that only the inner region cluster core has so far virialized. Giving each galaxy an average blue luminosity of $5.99 \times 10^9/h^2\ L_{B\odot}$, then yields the requisite total $1.95 \times 10^{12}/h^2\ L_{B\odot}$ surface blue luminosity within the Abell radius, to thus fully specify $I(R)$. Using as typical the mass to light ratio $M/L_B = 5.6hM_\odot/L_{B\odot}$ obtained for the galaxy NGC 3198 in Mannheim(1993) enables us to determine the mass volume density associated with $\sigma(r)$. It is very convenient to express this mass density in units of the standard critical density $\rho_c = 3H_0^2/8\pi G$, and we find that $\sigma(0') = 241.5\rho_c$, $\sigma(56.8') = \rho_c$, $\sigma(120') = 0.11\rho_c$, and $\sigma(185') = 0.03\rho_c$. The cluster is thus apparently merging with the general cosmological background at no more than $185'$ or so, and would be restricted to the first $57'$ if the density of the Universe is critical. Thus in a low density Universe we would put the edge of the cluster at $185'$, while in a high density one we would only consider the potentials of the first $57'$ of data as contributing to the velocity dispersion, with the next $128'$ of data then only contributing along with the rest of the galaxies in the Universe to the general Hubble flow. (Noting that the conventional estimation of the cosmological ratio ρ/ρ_c is made in comoving coordinates while our analysis here involves the same ratio in static coordinates, our determination of where the static cluster actually merges with the comoving background is thus perforce only a rough estimate.) Since the actual density of the Universe represents one of the key unknown issues in cosmology, we shall calculate cluster core virial velocities for both the high and low density Universe cases, and actually find below that the values that we then obtain turn out to be insensitive to where the cluster ends. Moreover, no matter where we put the edge of the cluster, we should not expect the matter within the cluster but close to this edge to necessarily be completely decoupled from the cosmological background, so that again only core virialization seems reasonable. However, since the modified Hubble is still a falling profile, it turns out that one quarter by volume of the entire $20R_0$ cluster is contained within $r \leq 2.5R_0$, while one half is contained within $r \leq 5R_0$. Thus virialization of only the core region is quite non-trivial.

Taking $V(r) = -\beta_{gal}c^2/r + \gamma_{gal}c^2r/2$ as the potential put out by a typical individual galaxy and taking N to be the total number of galaxies contained in the entire cluster (and not just the number $N(r_m)$ contained in the virialized $r \leq r_m$ region) fixes the overall normalization $(N\beta_{gal}c^2/R_0)^{1/2}$ of the Newtonian potential contribution to the cluster virial, while taking as typical the NGC 3198 gamma to light ratio of $9.2 \times 10^{-40}h^3/cm/L_{B\odot}$ also obtained in Mannheim(1993) then enables us to fix the overall normalization $(N\gamma_{gal}c^2R_0)^{1/2}$ of the linear potential contribution as well. Thus for a Coma cluster composed solely of luminous matter alone, the overall normalizations $(N\gamma_{gal}c^2R_0)^{1/2}$ and $(N\beta_{gal}c^2/R_0)^{1/2}$ needed for Eq. (4) take respective values of 10960 km/sec and 576 km/sec for a cluster cut off at $R_M = 20R_0$ ($N = 425$ galaxies). In the absence of any dark matter the luminous Newtonian contribution to the virial is thus negligible, while the linear contribution associated with the luminous matter is substantial. Specifically, if the entire $R_M = 20R_0$ cluster is virialized, Eq. (4) yields a virial velocity $\sigma_p(20R_0) = 10178$ km/sec, while also yielding partial virial velocities $\sigma_p(R_0) = 1089$ km/sec, $\sigma_p(1.5R_0) = 1678$ km/sec, $\sigma_p(2R_0) = 2195$ km/sec, and $\sigma_p(6.15R_0) = 5018$ km/sec in various inner regions. Similarly, if we cut off the cluster at $57' = 6.15R_0$ (to yield $N = 242$ galaxies, $(N\gamma_{gal}c^2R_0)^{1/2} = 8261$ km/sec, $(N\beta_{gal}c^2/R_0)^{1/2} = 435$ km/sec) we obtain the partial virial velocities $\sigma_p(R_0) = 1028$ km/sec, $\sigma_p(1.5R_0) = 1583$ km/sec, $\sigma_p(2R_0) = 2070$ km/sec, and $\sigma_p(6.15R_0) = 4885$ km/sec. The core region velocities are thus essentially insensitive to whether we use a high or low density Universe cut-off. From the line of sight velocity data points we find that the numerical average of all the points in $R \leq 1.3R_0$ is 1200 ± 195 km/sec, while that of the all the points in $R \leq 1.7R_0$ is 1185 ± 195 km/sec. However, before we assess the significance of these numbers, it is important to note that once less than the entire spherical cluster is virialized, then any given line of sight through the sphere, even those at small impact parameter R, will pass through both virialized and non-virialized regions (the integration in Eq. (2) is to R_M and not merely to r_m), so that the detected projected velocity at that R will include some non-virialized contributions as well. For instance, if $r \leq 2.5R_0$ is virialized, then out of a $20R_0$ cluster the percentage of line of sight material which involves unvirialized radii $r > 2.5R_0$ is 25% at $R = 1.5R_0$, 44% at $R = 2R_0$, and of course 100% at $R = 2.5R_0$. Thus even though the partial virial of Eq. (4) itself only involves integrating up to r_m, the very use of initial input raw velocity data to estimate a magnitude for a virialized $\sigma_p(r_m)$ in the $r < r_m$ region becomes suspect once the cluster is less than fully virialized. From our calculated virial velocities we see that conformal gravity would thus appear to have no difficulty accommodating a virialized inner cluster region of the order of $r_m \sim 1.5R_0$ without needing to invoke dark matter, and given the just noted limitation on the use of the input velocity data in partially virialized systems, the theory could possibly even accommodate up to $r_m \sim 2.5R_0$, a region which contains close to one quarter by volume of all of the matter in the entire $185'$ of the cluster. Moreover, given the relevant time scales which were discussed above, it would even appear to be quite reasonable to expect inner region virialization up to one or two scale lengths or so. While we would certainly not expect any larger a portion of the cluster to have yet virialized, a first principles determination of the two-body correlation function and of its potential impact on Eqs. (1) and (4) could nonetheless prove to be very instructive, and might possibly even turn out to be definitive for the theory. (It is also possible to test the conformal theory in a way which is actually insensitive to how big a fraction of the cluster has in fact virialized, viz. cluster gravitational lensing which responds to all the matter in the cluster virialized or not; thus a yet to be made study of the conformal theory predictions for lensing should eventually provide an independent and definitive way of testing the theory on whole cluster scales.) Other than this issue though, it would appear that, in the first instance at least, the conformal gravity theory is indeed capable of meeting the demands of cluster virial velocity data, with the linear potential theory thus readily being extendable from galactic scales up to the much larger ones associated with the virialized core regions of clusters of galaxies without encountering any major difficulty. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.

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